PROBLEMS

1.1 Express the following quantities using prefixes.

(a) 5000 g **(b) 2000000 W**

(c) 52x1Tt0kg **(d) 225x10^{-8s}**

{(a)5kg (b) 2 MW (c) 5.2 μg (d) 2.25μs }

Solution:

(a) 5000 g

= 5 x 1000g

= 5 x 1kg (Since 1000g = 1kg)

= 5kg

(b) 2000,000 w

=2 x 1000000

= 2 x 10^6W (10^{6} = 1 Mega)

= 2 MW

(c) 52 x 10^{-10} kg

= 5.2 x l0 x 10^{-10} kg

= 5.2 x 10^{-9} kg

= 5.2 x 10^{-9} x 1000g (Since 1 kg = 1000 g)

= 5.2 x 10^{-9} x 10^3 g

= 5.2 x 10^{-9} x 10^{-6} g

$= 5.2\ \mu g$ $(10^{-6} = 1 micro(\mu))$

(d) 225×10^{-8} s

$= 225 \times 10^{2} \times 10^{-8}$ s

$= 2.25 \times 10^{-6}$ s

$= 2.25\ \mu s$ $(10^{-6} = 1 micro(\mu))$

1.2 How do the prefixes micro, nano and pico relate to each other?

Solution:

As we know

micro = $\mu = 10^{-6}$

nano = n = 10^{-9}

pico = p = 10^{-12}

The relation between micro, nano and pico can be written as.

micro = 10^{-6}

nano = $10^{-6} \times 10^{-3} = 10^{-6}$ micro

pico = $10^{-6} \times 10^{-6} = 10^{-6}$ micro

1.3 Your hair grows at the rate of 1 mm per day. Find their growth rate in nm s^{-1}. (11.57 nm s^{-1})

Solution:

Growth rate Of hair in nm s^{-1} = Imm per day

Growth rate of hair in one day = $24 \times 60 \times 60$ s

(Since 1 mm 10^{-3} m and one day = $24 \times 60 \times 60$ s), hence

1 mm per day = I $\times 10^{-3}$ m $\times 1/24 \times 60 \times 60$ s

$= l \times 10^{-3}$ m x 1/8400 m s^{-1}

$= l \times 10^{-3}$ m x 0.00001157

$= l \times 10^{-3}$ m x 1157×10^{-8} ms^{-l}

$= 1157 \times 10^{-2}$m x $10^{-9}ms^{-1}$

$=11.57 \times 10^{-9}$ ms^{-1}

1 mm per day =11.57 nms^{-1}

(because 10^{-9} ms^{-1} = 1n ms^{-1}).

1.4 Rewrite the following In Standard form. (Scientific notation)

(a) 1168 x 10^{-27} **(b) 32 x 10^{5}**

(C) 725 x 10 kg^{-5} **(d) 0.02 x 10^{-8}**

{(a) 1.168x10^{-24} (b) 3.2x10^{6} (c) 7.25g (d) 2 x10^{-10} }

Solution: (a) 1168 x 10^{-27} = 1.168 x 10^{3} x 10^{-27} = 1.168 x 10^{-24}

(b) 32 x 10^{5} = 3.2 x 10^{1} x 10^{5} = 3.2 x 10^{6}

(C) 725 x 10^{-5} kg=7.25X 10^{2} x 10^{-5} kg = 7.25 x 10^{-3} kg

As (10^{-3} kg = 1g), therefore

7.25 x 10^{-3} kg = 7.25g

(d) 0.02 x 10^{-8} = 2 x 10^{-2} x 10^{-8} = 2 x 10^{-10}

1.5 Write the following quantities in standard form.

(a) 6400 km **(b) 38000 km**

(c) 300000000 ms^{-1} **(d) seconds in a day**

{(a) 6.4x10^{3} km (b) 3.8 x 10^{5} km (c) 3 x 10^{8} ms^{-1} (d) 0.64x10^{4}s)

Solution:

(a) 64000 km

Multiplying and dividing by "10^3"

$=6400$ m/1000 x 10^3km

$=64$ m/ 10 x 10^3km

$=6.4\times10^3$ km

(b) 38000 km

Multiplying and dividing by "10^5"

$=38000 / 10^5 \times 10^5$km

$=380000 / 10^50000 \times 10^5$km

$= 3.8 \times 10^5$ km

(c) 300000000 ms^{-1}

Multiplying and dividing by "10^8"

300000000 ms^{-1} / 100000000 x 10^8km

$= 3 \times 10^8$ km

(d) seconds in a day

As we know

1 day =24 hours

1 hour= 60 minutes

1 minute = 60 seconds so

1 day = 24 x 60 x 60 seconds

1 day = 86400 s

Multiplying and dividing by 10^4

=86400 / 10000 x 10^4 s

=8.4 x10^4 s

1.6 On closing the jaws Of a Vernier Calipers, zero of the Vernier sale is on the right to its main scale such that 4th division of its Vernier scale coincides with one of the main scale division. Find its zero error and zero correction. (+0.04cm, -0.04 cm)

Solution:

Main scale reading = 0.0 cm.

Vernier division coinciding with main scale = 4th division

Vernier scale reading = 4 x 0.01 cm = 0.04 cm

Zero error = 0.0 cm + 0.04 cm = 0.04 cm

Zero correction (Z.C) = -0.04 cm

The zero error of the Vernier scale is 0.04cm and its zero correction is -0.04cm

(Vernier division coinciding with main scale) = 4 div

Vernier scale reading = 4 x 0.01 cm

= 0.04 cm

Since zero of the Vernier scale is on the right side of the zero of the main scale, thus the instrument has measured more than the actual reading. It IS said to be positive zero error.

Zero correction is the negative of zero error. Thus

Zero error = +0.04 cm

and Zero correction = - 0.04 cm

1.7 A screw gauge has 50 divisions on its circular sale. The pitch of the screw gauge is 0.5 mm. What is its least count?
(0.001 cm)

Solution:

Number Of division on the circular scale = 50

Pitch of screw gauge = 0.5 mm

Least count Of screw gauge L.C. = ?

Least count = Pitch / Number Of division on the circular scale

Least count = 0.5mm / 50

= 0.01 mm = 0.01 x 1/10 cm

Least count = 0.001 cm

1.8 Which of the following quantities have three figures?

(a) 3.0066 m **(b) 0.00309 kg**

(c) 5.05x 10^{-27}kg **(d) 301.0 s**

{(b) and (c)}

Solution:

(a) 3.0066m

Zeros between significant digits are significant. Therefore, there are 5 significant figures in 3.0066m.

(b) 0.00309kg

Zeros used for spacing the decimal point are not significant. Therefore, there are 3 significant figures in 0.00309kg.

(C) 5.05 x 10^{-27}kg

Only the digits before the exponent are considered, thus there are 3 significant figures.

(d) 301.0s

Final zeros or zeros after the decimal are significant. Therefore, there are 4 significant figures.

Result:

Quantities (b) and (c) have three significant figures

1.9 What are the significant figures in the following measurements?

(a) 1.009 m **(b) 0.00450 kg**

(c) 1.66 x 10^{-27}kg **(d) 2001 s**

{(a) 4 (b) 3 (c) 3 (d) 4}

Solution:

(a) 1.009m

Since zeros between two significant figures are Significant, so there are 4 significant figures.

(b) 0.00450

Zeros used for spacing the decimal point are not significant. Hence, there are 3 significant figures.

(c) 1.66 x 10^{-27}kg

Only the digits before the exponent are considered so there are 3 significant figures.

(d) 2001s

Since zeros between two significant figures are significant so there are 3 significant figures.

1.10 A chocolate wrapper is 6.7 cm long and 5.4 cm wide. Calculate its area up to reasonable number of significant figures.
(36 cm^2)

Solution:

Length of chocolate wrapper l = 6.7 cm

Width of chocolate wrapper w = 5.4 cm

Area = A = ?

Area = Length x Width

A = l x w

$A = 6.7 cm \times 5.4 cm = 36.18 cm^2 = 36 cm^2$

Note:

Answer should be in two significant figures because in data the least significant figures are two therefore answer is 36 cm^2.

PROBLEMS

2.1 A train moves with a uniform velocity of 36 kmh-1 for 10 s. Find the distance travelled by it.
(100 m)

Solution: Velocity = V = 36 kmh^{-1} = 36 x 1000/ 60 x 60 = 36000/ 3600 = 10 ms^{-1}

Time t = 10 s

Distance = S =?

S = Vt

S = 10 x 10 = 100 m

2.2 A train starts from rest. It moves through 1 km in 100 s with uniform acceleration. What will be its speed at the end of 100 s.? (20 ms^{-1})

Solution: Initial velocity V_i = 0 ms^{-1}

Distance S = 1 km = 1000 m

Time = 100 s

Final velocity V_f =?

$S = V_i \times t + 1/2 \times a \times t2$

$1000 = 0 \times 100 + 1/2 \times a \times (100)^2$

$1000 = 1/2 \times 10000a$

$1000 = 5000a$

$a = 1000/5000 = 0.2\ ms^{-2}$

Now using 1st equation of motion

$V_f = V_i + at$

$V_f = 0 + 0.2 \times 100$

$V_f = 20\ ms^{-1}$

2.3 A car has a velocity of 10 ms^{-1}. It accelerates at 0.2 ms^{-2} for half minute. Find the distance travelled during this time and the final velocity of the car. (390 m, 16 ms^{-1})

Solution: Initial velocity = $V_i = 10\ ms^{-1}$

Acceleration $a = 0.2\ ms^{-2}$

Time $t = 0.5$ min $= 0.5 \times 60 = 30$ s

(I) Distance S =?

(II) Final velocity V_f =?

$S = V_i \times t + 1/2 \times a \times t^2$

$S = 10 \times 30 + 1/2 \times 0.2 \times (30)^2$

$S = 300 + 1/2 \times 0.2 \times 90$

$S = 300 + 1/2 \times 2/10 \times 90$

$S = 300 + 90$

$S = 390$ m

(II) Using 1st equation of motion

$V_f = V_i + at$

$V_f = 10 + 0.2 \times 30$

$V_f = 10 + 6$

$V_f = 16\ ms^{-1}$

2.4 A tennis ball is hit vertically upward with a velocity of 30 ms^{-1}, it takes 3 s to reach the highest point. Calculate the maximum height reached by the ball. How long it will take to return to ground? (45 m, 6 s)

Solution: Initial velocity = $V_i = 30\ ms^{-1}$

Acceleration due to gravity $g = -10\ ms^{-2}$

Time to reach maximum height = $t = 3\ s$

Final velocity $V_f = 0\ ms^{-1}$

(I) Maximum height attained by the ball S =?

(II) Time taken to return to ground t =?

$$\mathbf{S = V_i \times t + 1/2 \times g \times t^2}$$

$S = 30 \times 3 + 1/2 \times (-10) \times (3)^2$

$S = 90 - 5 \times 9$

$S = 90 - 45$

$S = 45\ m$

Total time = time to reach maximum height + time to return to the ground

= 3 s + 3 s = 6 s

2.5 A car moves with uniform velocity of 40 ms^{-1} for 5 s. It comes to rest in the next 10 s with uniform deceleration.

Find:

(I) deceleration
(II) total distance travelled by the car.

(-4 ms^{-2}, 400 m)

Solution: Initial velocity = Vi = 40 ms^{-1}

Time = t = 5 s

Final velocity = Vf = 0 ms^{-1}

Time = 10 s

(I) deceleration a =?

(II) total distance S =?
$V_f = V_i + at$

Or $at = V_f - V_i$

$a = V_f - V_i/t$

$a = 0 - 40/10$

$a = -4\ ms^{-2}$

Total distance travelled = $S = S_1 + S_2$

By using this relation

$S_1 = V_t$

$S_1 = 40 \times 5$

$S_1 = 200\ m$ .... (i)

Now by using 3rd equation of motion

$2aS_2 = V_f^2 - V_i^2$

$S_2 = V_f^2 - V_i^2/2a$

$S_2 = (0)2 - (40)2/2 \times (-4)$

$S_2 = -1600/-8$

$S_2 = 200\ m$ .... (ii)

From (i) and (ii) we get;

$S = S_1 + S_2$

Or $S = 200\ m + 200\ m$

$S = 400\ m$

2.6 A train starts from rest with an acceleration of 0.5 ms^{-2}. Find its speed in kmh-1, when it has moved through 100 m. (36 kmh^{-1})

Solution: Initial velocity $V_i = 0\ ms^{-1}$

Acceleration $a = 0.5\ ms^{-2}$

Distance $S = 100\ m$

Final velocity $V_f = ?$

$2aS = Vf^2 - V_i^2$

$2 \times 0.5 \times 100 = Vf^2 - 0$

Or , $100 = Vf^2$

Or $Vf^2 = 100\ ms^{-1}$..........................(I)

Speed in kmh^{-1}:

From (I) we get;

$V_f = 10 \times 3600/1000 = 36\ kmh^{-1}$

2.7 A train starting from rest, accelerates uniformly and attains a velocity 48 kmh^{-1} in 2 minutes. It travels at this speed for 5 minutes. Finally, it moves with uniform retardation and is stopped after 3 minutes. Find the total distance travelled by the train.

Solution: Case – I: **(6000 m)**

Initial velocity = $V_i = 0ms^{-1}$

Time = t = 2 minutes = 2x 60 = 120 s

Final velocity = $V_f = 48\ kmh^{-1} = 48 \times 1000/3600 = 13.333\ ms^{-1}$

$S_1 = V_{av} \times t$

$S_1 = (V_f + V_i)/2 \times t$

$S_1 = 13.333 + 0/2 \times 120$

$S_1 = 6.6665 \times 120$

$S_1 = 799.99\ m = 800\ m$

Case – II:

Uniform velocity = V_f = 13.333 ms^{-1}

Time = t = 5 minutes = 5 x 60 = 300 s

$S_2 = v \times t$

$S_2 = 13.333 \times 300$

$S_2 = 3999.9 = 4000\ m$

Case – III:

Initial velocity = V_f = 13.333 ms^{-1}

Final velocity = V_i = 0 ms^{-1}

Time = t = 3 minutes = 3 x 60 = 180 s

$S_3 = V_{av} \times t$

$S_3 = (V_f + V_i)/2 \times 180$

$S_3 = 13.333 + 0/2 \times 180$

$S_3 = 6.6665 \times 180$

$S_3 = 1199.97 = 1200\ m$

Total distance = $S = S_1 + S_2 + S_3$

$S = 800 + 4000 + 1200$

$S = 6000\ m$

2.8 A cricket ball is hit vertically upwards and returns to ground 6 s later. Calculate

(i) Maximum height reached by the ball

(ii) initial velocity of the ball (45m, 30 ms-1)

Solution: Acceleration due to gravity = g = -10 ms^{-1} (for upward motion)

Time to reach maximum height (one sided time) = t = 6/2 = 3 s

Velocity at maximum height = V_f = 0 ms^{-1}

(i) Maximum height reached by the ball S = h =?
(ii) Maximum initial velocity of the ball = V_i =?

Since, $V_f = V_i + gxt$

$V_i = V_f - gxt$

$V_i = 0 - (-10) \times 3$

$V_i = 30\ ms^{-1}$

Now using 3rd equation of motion

$2aS = V_{f2} - V_{i2}$

$S = V_{f2} - V_{i2}/2a$

$S = (0)2 - (30)2/2 \times (-10)$

$S = -90/-20$

$S = 45$ m

2.9 When brakes are applied, the speed of a train decreases from 96 kmh-1 to 48 kmh-1 in 800 m. How much further will the train move before coming to rest? (Assuming the retardation to be constant).
(266.66 m)

Solution: Initial velocity = V_i = 96 kmh^{-1} = 96 x 1000/3600 = 96000/3600 ms^{-1}

Final velocity = V_f = 48 kmh^{-1} = 48 x 1000/3600 = 48000/3600 ms^{-1}

Distance = S = 800 m

Further Distance = S_1 =?

First of all, we will find the value of acceleration a

$2aS = V_f^2 - V_i^2$

$2 \times a \times 800 = (48000/3600)^2 - (96000/3600)^2$

$1600a = (48000/3600)^2 - ((2 \times 48000)/3600)^2$ (96000 = 2 x 48000)

$1600a = (48000/3600)^2 ((1)^2 - (2)^2)$ (taking (48000/36000) as common)

$1600a = (48000/3600)^2 (1 - 4)$

$1600a = (48000/3600)^2 (-3)$

$a = (48000/3600)^2 \times 3/1600$

now, we will find the value of further distance S^2:

$V_f = 0$, S_2 =?

$2aS = V_f^2 - V_i^2$

$-2 (48000/36000)^2 \times 3/1600 \times S_1 = (0)^2 - (48000/36000)^2$

$S_1 = (48000/36000)^2 \times (48000/36000)^2 \times 1600/3 \times 2$

$S_1 = 1600/6$

$S_2 = 266.66m$

2.10 In the above problem, find the time taken by the train to stop after the application of brakes.
(80 s)

Solution: by taking the data from problem 2.9:

Initial velocity = V_i = 96 kmh^{-1} = 96 x 1000/3600 = 96000/3600 ms^{-1}

Final velocity = V_f = 0 ms^{-1}

a = -(48000/3600)^2 x 3/1600 ms^{-2}

time = t =?

Or $V_f = V_i + at$

Or $at = V_f - V_i$

$t = V_f - V_i/a$

t = 0 - (48000/3600)/-((48000/3600) x 1/1600

t = - 96000/3600 x (3600/48000)^2 x 1600/3

t = 2 x 48000/3600 x (3600/48000 x 3600/48000) x 1600/3

t = 2 x 3600/3 x 3

t = 2 x 40 = 80 s

Numerical

3.1 A force of 20 N moves a body with an acceleration of 2 ms^{-2}. What is its mass?

(10kg)

Solution: Force = F = 20 N

Acceleration = a = 2 ms^{-2}

Mass = m = **?**

F = ma

Or $m = \frac{F}{a}$

$m = \frac{20}{2}$ = 10 kg

3.2 The weight of a body is 147 N. What is its mass? (Take the value of g as 10 ms^{-2})

(14.7 kg)

Solution: Weight = w = 147 N

Acceleration due to gravity = g = 10 ms^{-2}

Mass = m = **?**

w = mg

or $m = \frac{w}{g}$

$m = \frac{147}{10}$

$m = 14.7\ kg$

3.3 How much force is needed to prevent a body of mass 10 kg from falling?

(100 N)

Solution: Mass = m = 50 kg

Acceleration = a = g = 10 ms $^{-2}$

Force = F = **?**

F = m a

F = 10 x 10

F = 100 N

3.4 Find the acceleration produced by a force of 100 N in a mass of 50 kg.

(2 ms $^{-2}$)

Solution: Force = F = 100 N

Mass = m = 50 kg

Acceleration = a = **?**

F = m a

Or a = $\frac{F}{m}$

a = $\frac{100}{50}$

a = 2 ms $^{-2}$

3.5 A body has weight 20 N. How much force is required to move it vertically upward with an acceleration of 2 ms^{-2}?

(24 N)

Solution: Weight = w = 20 N

Acceleration = a = 2 ms^{-2}

Vertically upward force (Tension) = T = **?**

F_{net} = T – w

Or ma = T – mg

Or ma + mg = T

Or T = m (a + g) ....(i)

Now, $m = \frac{w}{g}$

$m = \frac{20}{10} = 2$ kg

Putting the value of m in Eq.(i), we get

T = 2 (2 + 10)

= 2(12)

T = 24 N

3.6 Two masses 52 kg and 48 kg are attached to the ends of a string that passes over a frictionless pulley. Find the tension in the string and acceleration in the bodies when both the masses are moving vertically.

(500 N, 0.4 ms^{-2})

Solution: m_1 = 52 kg and m_2 = 48 kg

(i) Tension T = ?

(ii) Acceleration a = ?

(i) $T = \dfrac{2m_1 m_2}{m_1 + m_2}\, g$

$T = \dfrac{2 \times 52 \times 48}{52 + 48} \times 10$

$T = \dfrac{49920}{100}$

$T = 499.20 \approx 500$ N

(ii) $a = \dfrac{m_1 \quad m_2}{m_1 + m_2}\, g$

$a = \dfrac{52 - 48}{52 + 48} \times 10$

$a = \dfrac{4}{100} \times 10$

$a = 0.4\ ms^{-2}$

3.7 Two masses 26 kg and 24 kg are attached to the ends of a string which passes over a frictionless pulley. 26 kg is lying over a smooth horizontal table. 24 N mass is moving vertically downward. Find the tension in the string and the acceleration in the bodies.

(125 N, 4.8 ms^{-2})

Solution: : $m_1 = 24$ kg and $m_2 = 26$ kg

(i) Tension $T = ?$

(ii) Acceleration $a = ?$

(i) $T = \dfrac{m_1 m_2}{m_1 + m_2}\, g$

$T = \dfrac{24 \times 26}{24 + 26} \times 10$

$T = \dfrac{6240}{50}$

$T = 124.8 \approx 125$ N

(ii) $$a = \frac{m_1}{m_1 + m_2} g$$

$$a = \frac{24}{24 + 26} \times 10$$

$$a = \frac{24}{50} \times 10$$

$$a = 4.8 \text{ ms}^{-2}$$

3.8 How much time is required to change 22 Ns momentum by a force of 20 N?

(1.1s)

Solution: Change in momentum = $P_f - P_i$ = 22 Ns

Force = F = 20 N

Time = t = **?**

$$F = \frac{P_f \quad P_i}{t}$$

$$t = \frac{P_f \quad P_i}{F}$$

$$t = \frac{22}{20} = 1.1 \text{ S}$$

3.9 How much is the force of friction between a wooden block of mass 5 kg and the horizontal marble floor? The coefficient of friction between wood and the marble is 0.6.

(30 N)

Solution: Mass = m = 5 kg

Coefficient of friction = μ = 0.6

Force of friction = F_S = **?**

$F_S = \mu R$ (where R = mg)

$F_S = \mu mg$

$F_S = 0.6 \times 5 \times 10 = 30$ N

3.10 How much centripetal force is needed to make a body of mass 0.5 kg to move in a circle of radius 50 cm with a speed 3 ms $^{-1}$?

(9 N)

Solution: Mass = m = 0.5 kg

Radius of the circle = r = 50 cm = $\frac{50}{100}$ = 0.5 m

Speed = v = 3 ms $^{-1}$

Centripetal force = F_c = **?**

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{0.5 \times 3^2}{0.5}$$

$$F_c = \frac{0.5 \times 9}{0.5} = \frac{4.5}{0.5} = 9\ N$$

PROBLEMS

4.1 Find the resultant of the following forces:

(i) 10 N along x-axis

(ii) 6 N along y-axis and

(iii)4 N along negative x-axis. (8.5 N making 45° with x-axis)

Solution: F_x = Net force along x-axis = 10.4 = 6 N

F_y = Force along y-axis = 5 N

Magnitude of the resultant force = F = ?

$F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{(6)^2 + (6)^2}$

$F = \sqrt{36 + 36}$

$= \sqrt{72} = 8.5$ N

Now, $\theta = \tan^{-1} = F_y / F_x$

$\theta = \tan^{-1} = 6 / 6$

$\theta = \tan^{-1} (1)$

$\theta = 45$ with x-axis

4.2 Find the perpendicular components of a force of 50 N making an angle of 30° with x-axis. (43.3 N, 25 N)

Solution: Force F = 50 N

Angle $\theta = 30$

$F_x = ?$ and $F_y = ?$

$F_x = F \cos \theta$

$F_x = 50 \times \cos 30$

$= 50\text{ N} \times 0.866$ $(\because \cos 30 = 0.866)$

$F_x = 43.3\text{ N}$

Similarly, $F_y = F \sin \theta$

$F_y = 50 \times 0.5$ $(\because \sin 30 = 0.5)$

$F_y = 25\text{ N}$

4.3 Find the magnitude and direction of a force, if its x-component is 12 N and y-component is 5 N. (13 N making 22.6° with x-axis)

Solution: $F_x = 12\text{ N}$

$F_y = 5\text{ N}$

(i) Magnitude of the force = F = ?
(ii) Direction of the force = θ = ?

$F = \sqrt{F_x^2 + F_y^2}$
$F = \sqrt{(12)^2 + (5)^2}$
$F = \sqrt{144+25}$
$F = 13\text{ N}$

(ii) $\theta = \tan^{-1} = F_y / F_x$

$\theta = \tan^{-1} = 12 / 5$

$\theta = \tan^{-1} (2.4)$

$\theta = 22.6$ with x-axis

4.4 A force of 100 N is applied perpendicularly on a spanner at a distance of 10 cm from a nut. Find the torque produced by the force. (10 Nm)

Solution: Force = F = 100 N

Distance = L = 10 cm = 0.1 m

Torque T = ?

Torque T = F × L

= 100 N × 0.1 m

= 10 Nm

4.5 A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. (23.1 N)

Solution: Angle θ = 30 (with x-axis)

Horizontal component of force F_x = 20 N

Force F = ?

$F_x = F \cos\theta$

20 N = F cos 30

20 N = F × 0.866 (∵ cos 30 = 0.866)

F = 20 N / 0.866 = 23.09

F = 23.1 N

4.6 The steering of a car has a radius 16 cm. Find the torque produced by a couple of 50 N. (16 Nm)

Solution: Radius = r = L = 16 cm = 16/100 m = 0.16 m

Couple arm = L = 16 cm = 16/100 m = 0.16 m

Force = F = 50 N

Torque T = ?

Torque T = F × L

$= 50\ N \times (2 \times 0.16)$

$= 16\ Nm$

4.7 A picture frame is hanging by two vertical strings. The tensions in the strings are 3.8 N and 4.4 N. Find the weight of the picture frame. (8.2 N)

Solution: Tension $T_1 = 3.8$ N

Tension $T_2 = 4.4$ N

Weight of the picture frame = w = ?

When the picture is in equilibrium, then

$\sum F_x = 0$ and $\sum F_y = 0$

Therefore $T - w = 0$

Or $(T_1 + T_2) - w = 0$

$T_1 + T_2 = w$

$3.8 + 4.4 = w$

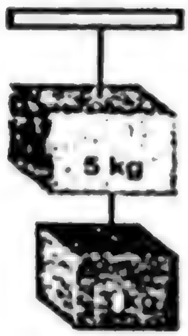
$W = 8.2$ N

4.8 Two blocks of mass 5 kg and 3 kg are suspended by the two strings as shown. Find the tension in each string. (80 N, 30 N)

Solution: Mass of large block = M = 5 kg

Mass of large block = m = 3 kg

Tension produced in each string = T_1 = ? and T_2 = ?



$T_1 = w_1 + w_2$

$T_1 = Mg + mg$

$T_1 = (M + m)g$

$T_1 = (3+5) \times 10$

$= 8 \times 10$

$= 80$ N

Also, $T_2 = mg$

$T_2 = 3 \times 10 = 30$ N

4.9 A nut has been tightened by a force of 200 N using 10 cm long spanner. What length of a spanner is required to loosen the same nut with 150 N force? (13.3 cm)

Solution: Force $= F_1 = 200$ N

Length $= L_1 = 10$ cm $= 10 / 100 = 0.1$ m

Length of the spanner to tighten the same nut:

Force $= F_2 = 150$ N

Length $= L_2 = ?$

Since $T_1 = T_2$

$F_1 \times L_1 = F_2 \times L_2$

$200 \times 0.1 = 150 \times L_2$

$20 = 150 \times L_2$

$L_2 = 20 / 150 = 0.133$ m $= 0.133 \times 100 = 13.3$ cm

4.10 A block of mass 10 kg is suspended at a distance of 20 cm from the center of a uniform bar 1 m long. What force is required to balance it at its center of gravity by applying the force at the other end of the bar? (40 N)

Solution: Mass of the block = m = 10 kg

Length of the bar = l = 1 m

Moment arm of $w_1 = L_1 = 20$ cm = 0.2 m

Moment arm of $w_2 = L_2 = 50$ cm = 0.5 m

Force required to balance the bar F_2 = ?

By applying principle of moments:

Clockwise moments = anticlockwise moments

$F_1 \times L_1 = F_2 \times L_2$

$mg \times L_1 = F_2 \times L_2$

$(10 \times 10) \times 0.2 = F_2 \times 0.5$

$20 = F_2 \times 0.5$

$F_2 = 20 / 0.5 = 200 / 5 = 40$ N

PROBLEMS

5.1 Find the gravitational force of attraction between two spheres each of mass 1000 kg. The distance between the centers of the spheres is 0.5 m.

(2.67×10^{-4} N)

Solution: Mass = $m_1 = m_2 = 1000$ kg

Distance between the centers = d = 0.5 m

Gravitational constant = G = 6.673×10^{-11} Nm^2kg^{-2}

Gravitational force = F = ?

$$F = \frac{G\, m_1 m_2}{d_2}$$

$$F = 6.673 \times (10)^{-11} \times \frac{1000 \times 1000}{(0.5)^2}$$

$$= 6.673 \times (10)^{-11} \times \frac{(10)^6}{0.25}$$

$$= \frac{6.673 \times (10)^{-11} \times (10)^6}{0.25}$$

$$= \frac{6.673 \times (10)^{-5}}{0.25}$$

$$= 26.692 \times (10)^{-5} = 2.67 \times 10^{-4} \text{ N}$$

5.2 The gravitational force between two identical lead spheres kept at 1 m apart is 0.006673 N. Find their masses. (10, 000 kg each)

Solution: Gravitational force = F = 0.006673 N

Gravitational constant = G = 6.673×10^{-11} Nm^2kg^{-2}

Distance between the masses = d = 1m

Mass = $m_1 = m_2 = ?$

$F = G\frac{m_1 m_2}{d_2}$

$F = G\frac{m \times m}{d_2}$ (Let $m_1 = m_2 = m$)

$F = \frac{m^2}{d_2}$

$m^2 = \frac{F \times d^2}{G}$

$m^2 = \frac{0.006673 \times (1)^2}{6.673 \times (10)^{11}} = \frac{\frac{6673}{1000000}}{6.673 \times (10)^{11}}$

$= \frac{6.673 \times (10)^3}{6.673 \times (10)^{11}}$

$\sqrt{m^2} = 10^8$

$m = 10^4 = 10000$ kg each

Therefore, mass of each lead sphere is 10000 kg.

5.3 Find the acceleration due to gravity on the surface of the Mars. The mass of Mars is 6.42×10^{23} kg and its radius is 3370 km.

Solution: Mass of Mars = $M_m = 6.42 \times 10^{23}$ kg

Radius of Mars = R_m = 3370 km = 3370 × 1000 m = 3.37×10^8 m

Acceleration due to gravity of the surface of Mars = $g_m = ?$

$g_m = G\frac{M_m}{R^2{}_m}$

or $g_m = 6.673 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{(3.37 \times 10^6)^2}$

$= \frac{6.673 \times 10^{11} \times 6.42 \times 10^{23}}{11.357}$

$= \frac{42.84}{11.357} = 3.77\ m^{-2}$

5.4 The acceleration due to gravity on the surface of moon is 1.62 ms^{-2}. The radius of Moon is 1740 km. Find the mass of moon.

Solution: Acceleration due to gravity = g_m = 1.62 ms^{-2}

Radius of the moon = R_m = 1740 km = 1740 × 1000 m = 1.74×10^6 m

Mass of moon = M_m = ?

$$g_m = G\frac{M_m}{R^2{}_m}$$

or $$M_m = \frac{gm \times R^2{}_m}{G}$$

$$M_m = \frac{1.62 \times (1.74 \times 10^6)^2}{6.673 \times 10^{\ \ 11}}$$

$$= \frac{1.62 \times 3 \times 10^{12}}{6.673 \times 10^{\ \ 11}}$$

$$= \frac{4.86 \times 10^{12} \times 10^{11}}{6.673}$$

$M_m = 7.35 \times 10^{22}$ kg

5.5 Calculate the value of g at a height of 3600 km above the surface of the Earth. (4.0 ms^{-2})

Solution: Height = h = 3600 km = 3600 × 1000 m = 3.6×10^6 m

Mass of Earth = M_e = 6.0×10^{24} kg

Gravitational acceleration = g_h =?

$$g_h = \frac{GM_e}{(R+h)^2}$$

$$g_h = 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(6.4 \times 10^6 + 3.6 \times 10^6)^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(10.0 \times 10^6)^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{100 \times 10^{12}}$$

$$= 6.673 \times 10^{-11} \times 6.0 \times 10^{10} = 40 \times 10^{-1} = 4.0\ ms^{-2}$$

5.6 Find the value of g due to the Earth at geostationary satellite. The radius of the geostationary orbit is 48700 km. ($0.17\ ms^{-2}$)

Solution: Radius = R = 48700 × 1000 m = 4.87×10^7 m

Gravitational acceleration = g =?

$$g = \frac{GM_e}{R^2}$$

$$g = 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(4.87 \times 10^{7})^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(23.717 \times 10^{14})^2}$$

$$= \frac{6.673 \times 6.0 \times 10^{\ 1}}{23.717} = \frac{4.0038}{23.717}$$

$$= 0.17\ ms^{-2}$$

5.7 The value of g is $4.0\ ms^{-2}$ at a distance of 10000 km from the center of the Earth. Find the mass of the Earth. (5.99×10^{24} kg)

Solution: Gravitational acceleration = g = $4.0\ ms^{-2}$

Radius of Earth = R_e = 10000 km = 10000 × 1000 m = 10^7 m

Mass of Earth = M_e =?

$$M_e = \frac{gR^2}{G}$$

$$M_e = \frac{4.0 \times (10^7)^2}{6.673 \times 10^{\ 11}}$$

$$= \frac{4.0 \times 10^{14}}{6.673 \times 10^{\ 11}}$$

$$= 0.599 \times 10^{25}\ kg = 5.99 \times 10^{24}\ kg$$

5.8 At what altitude the value of g would become one fourth than on the surface of the Earth? (One Earth's radius)

Solution: Mass of Earth = $M_e = 6.0 \times 10^{24}$ kg

Radius of Earth = $R_e = 6.4 \times 10^6$ m

Gravitational acceleration = $g_h = \frac{1}{4} g = \frac{1}{4} \times 10 \text{ ms}^{-2} = 2.5 \text{ ms}^{-2}$

Altitude above Earth's surface = h =?

$$g_h = \frac{GM_e}{(R+h)^2}$$

or $(R + h)^2 = \frac{GM_e}{g_h}$

Taking square root on both sides

or $\sqrt{(R + h)^2} = \sqrt{\frac{GM_e}{g_h}}$

or $R + h = \sqrt{G\frac{GM_e}{g_h}}$

or $h = \sqrt{G\frac{GM_e}{g_h} - R}$

or $h = \sqrt{\frac{6.673 \times 10^{\ \ 11} \times 6.0 \times 10^{24}}{2.5}} - 6.4 \times 10^6$

$= \sqrt{\frac{40.038 \times 10^{13}}{2.5}} - 6.4 \times 10^6$

$= \sqrt{16 \times 10^{13} \text{m}^2} - 6.4 \times 10^6 = \sqrt{0.16 \times 10^{12}} - 6.4 \times 10^6$

$= 0.4 \times 10^6 - 6.4 \times 10^6$

$= -6.0 \times 10^6$ m

As height is always taken as positive, therefore

$h = 6.0 \times 10^6$ m = One Earth's radius

5.9 A polar satellite is launched at 850 km above Earth. Find its orbital speed.

(7431 ms^{-1})

Solution: Height = h = 850 km = 850 × 1000 m = 0.85 × 10^6 m

Orbital velocity = v_o =?

$$v_o = \sqrt{\frac{GM_e}{R+h}}$$

$$v_o = \sqrt{\frac{6.673 \times 10^{\ 11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 0.85 \times 10^6}} = \sqrt{\frac{40.038 \times 10^{13}}{7.25 \times 10^6}}$$

$$= \sqrt{5.55 \times 10^7} = \sqrt{5.55 \times 10^6}$$

$$= 7.431 \times 10^3 = 7431\ ms^{-1}$$

5.10 A communication is launched at 42000 km above Earth. Find its orbital speed. (2876 ms^{-1})

Solution: Height = h = 42000 km = 42000 × 1000 m = 42 × 10^6 m

Orbital velocity = v_o =?

$$v_o = \sqrt{\frac{GM_e}{R+h}}$$

$$v_o = \sqrt{\frac{6.673 \times 10^{\ 11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 42 \times 10^6}}$$

$$= \sqrt{\frac{40.038 \times 10^{13}}{48.4 \times 10^6}}$$

$$= \sqrt{\frac{400.38 \times 10^{12}}{48.4 \times 10^6}}$$

$$= \sqrt{8.27 \times 10^6}$$

$$= 2.876 \times 10^3 = 2876\ ms^{-1}$$

Numerical Problems

6.1. A man has pulled a cart through 35 m applying a force of 300 N. Find the work done by the man. (10500 J)

Solution: Distance = S = 35 m

Force = F = 300 N

Work done = W = ?

W = F x S

W = 300 x 35

W = 10500 J

6.2. A block weighing 20 N is lifted 6 m vertically upward. Calculate the potential energy stored in it. (120 J)

Solution: Weight of the block = W = 20 N

Height = h = 6 m

Potential energy = P. E. = ?

P.E. = mgh

We know that w = mg

P.E. = (mg) x h

Thus, P.E. = (2 x 10) x 6

P.E. = 120 J

6.3. A car weighing 12 k N has speed of 20 ms – 1. Find its kinetic energy.

(240 kJ)

Solution: Weight of the car = w = 12kN = 12 x 1000 N = 12000 N

Speed of the car = v= 20 ms^{-1}

Kinetic energy = K.E. = ?

$$K.E. = \frac{1}{2} m v^2$$

$$W = mg \quad \text{or} \quad m = \frac{w}{g}$$

$$m = \frac{12000}{10} = 1200 \text{ kg}$$

Thus $$K.E. = \frac{1}{2} \times 1200 \times (20)^2$$

$$= 600 \times 400 = 240000 \text{ J}$$

$$K.E. = 240 \text{ kJ}$$

6.4. A 500 g stone is thrown up with a velocity of 15ms^{-1}. Find its

(i) P.E. at its maximum height

(ii) K.E. when it hits the ground

(56.25 J, 56.25 J)

Solution: Mass of stone = m = 500 g = $\frac{500}{1000}$ kg = 0.5 kg

Velocity = v = 15 ms^{-1}

(i) Potential energy = P.E. = ?

(ii) Kinetic energy = K.E. = ?

(i) Loss of K.E. = Gain in P.E.

$$\frac{1}{2} m \, v_f^2 - \frac{1}{2} m \, v_i^2 = mgh$$

As velocity of the stone at maximum height become zero, therefore, $v_f = 0$

$$\frac{1}{2} \times 0.5 \times (0) - \frac{1}{2} \times 0.5 \times (15)^2 = mgh$$

$$\frac{1}{2} \times 0.5 \times 225 = mgh$$

$$-56.25 = mgh$$

$$mgh = -56.25 J$$

Since energy is always positive, therefore

$$P.E. = 56.25 \text{ J}$$

(ii) $$K.E. = \frac{1}{2} m v^2$$

$$K.E. = \frac{1}{2} \times 0.5 \times (15)^2$$

$$= \frac{1}{2} \times 0.5 \times 225$$

$$= 56.25 \text{ J}$$

6.5. On reaching the top of a slope 6 m high from its bottom, a cyclist has a speed of 1.5 ms^{-1}. Find the kinetic energy and the potential energy of the cyclist. The mass of the cyclist and his bicycle is 40 kg. (45 J, 2400 J)

Solution: Height of the slope = h = 6 m

Speed of the cyclist = v = 1.5 ms^{-1}

Mass of cyclist and the bicycle = m = 40 kg

(i) Kinetic energy = K.E. = ?

(ii) Potential energy = P.E. = ?

(i) $$K.E. = \frac{1}{2} m v^2$$

$$K.E = \frac{1}{2} \times 40 \times (1.5)^2$$

$$K.E = \frac{1}{2} \times 40 \times 2.25 = 45 \text{ J}$$

(ii) $$P.E = mgh$$

$$P.E = 40 \times 10 \times 6 = 2400 \text{ J}$$

6.6. A motor boat moves at a steady speed of 4 ms^{-1}. Water resistance acting on it is 4000 N. Calculate the power of its engine.

(16 kW)

Solution: Speed of the boat = v = 4ms^{-1}

Force = F = 4000 N

Power = P = ?

$$P = F v$$

$$P = 4000 \times 4$$

$$P = 16000 \text{ W}$$

$$P = 16 \times 10^3 \text{ W}$$

$$P = 16 \text{ kW}$$

6.7. A man pulls a block with a force of 300 N through 50 m in 60 s. Find the power used by him to pull the block.

(250 W)

Solution: Force = F = 300 N

Distance = S = 50m

Time = t = 60 s

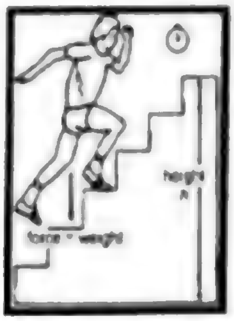
Power = P = ?

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{w}{t} = \frac{F \times S}{t}$$

$$P = \frac{300 \times 50}{60} = 5 \times 50 = 250 \text{ W}$$

6.8 A 50 kg man moved 25 steps up in 20 seconds. Find his power, if each step is 16 cm high.

(100 W)



Solution:

Mass = m = 50 kg

Total height = h = 25 x 16 = 400 cm = $\frac{400}{100}$ m = 4 m

Time = t = 20 s

Power = P = ?

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{w}{t} = \frac{mgh}{t}$$

$$P = \frac{50 \times 10 \times 4}{20}$$

$$P = 100 \text{ W}$$

6.9. Calculate the power of a pump which can lift 200 kg of water through a height of 6 m in 10 seconds.

(1200 watts)

Solution: Mass = m = 200 kg

Height = h = 6m

Power = P = ?

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{w}{t} = \frac{mgh}{t}$$

$$P = \frac{200 \times 10 \times 6}{10}$$

$$P = 1200 \text{ W}$$

6.10. An electric motor of 1hp is used to run water pump. The water pump takes 10 minutes to fill an overhead tank. The tank has a capacity of 800 liters and height of 15 m. Find the actual work done by the electric motor to fill the tank. Also find the efficiency of the system. (Density of water = 1000 kgm^{-3})

(Mass of 1 liter of water = 1 kg) (447600 J, 26.8 %)

Solution: Power = P = 1 hp = 746 W

Time = t = 10 min = 10 x 60 s = 600 s

Capacity/volume = V = 800 liters

Height = h = 15 m

(i) Work done = W = ?

(ii) Efficiency = E = ?

(i) Power = $\frac{\textbf{work}}{\textbf{time}}$

$$P = \frac{w}{t}$$

Or $W = P \times t$

$W = 746 \times 600$

$W = 447600$ J

Since the work done by electric pump to fill the tank is 447600 J. It is equal to input.

Hence input = actual work doe = W = 447600 J

(ii) Output = P.E = mgh

Since 1 litre = 1kg, therefore 800 litres = 800 kg

Output = P.E = 800 x 10 x15 = 120000J

$$\% \text{ Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100$$

$$\% \text{ Efficiency} = \frac{120000}{447600} \times 100$$

Efficiency = 26. 8 %

Numerical Problems

7.1 A wooden block measuring 40 cm x 10 cm x 5 cm has a mass 850 g. Find the density of 3 wood.

(425 kgm $^{-3}$)

Solution: volume of wooden block = V = 40 cm x 10 cm x 5cm = 2000 cm^3

$$= 2000 \times \frac{1}{100} \times \frac{1}{100} \times \frac{1}{100} \text{ m}^3$$

$$= 0.002 \text{ m}^3$$

$$\text{Mass} = m = 850 \text{ g} = \frac{850}{1000} = 0.85 \text{ kg}$$

Density of wood = ρ = ?

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{0.85}{0.02} = 425 \text{ kg}^{-3}$$

7.2 How much would be the volume of ice formed by freezing 1 litre of water?

(1.09 litre)

Solution: Volume of water = 1 litre

Volume of ice = ?

1 litre of water = 1 kg mass and density = 1000 kg $^{-3}$

Since density of ice is 0.92 times of the liquid water therefore,

$$\text{Volume of ice} = \frac{Mass}{Density}$$

$$= \frac{1000}{920}$$

Volume of ice = 1.09 litre

7.3 Calculate the volume of the following objects:

(i) An iron sphere of mass 5 kg, the density of iron is 8200 kgm $^{-3}$.

(6.1×10^{-4} m ^{3})

(ii) 200 g of lead shot having density 11300 kgm $^{-3}$.

(1.77×10^{-5} m ^{3})

(iii) A gold bar of mass 0.2 kg. The density of gold is 19300 kgm $^{-3}$.

(1.04×10^{-5} m ^{3})

Solution: Mass of iron sphere = m = 5 kg

Density of iron = ρ = 8200 kgm $^{-3}$

Volume of iron sphere = V =?

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Volume} = \frac{5}{8200}$$

$$= 0.00060975 = 6.0975 \times 10^{-4}$$

Volume = 6.1×10^{-4}m ^{3}

(ii) Mass of lead shot = m = 200 g = $\frac{200}{1000}$ kg = 0.2 kg

Density of lead = ρ = 11300 kgm $^{-3}$

Volume of lead shot = V =?

$$\text{Volume} = \frac{\textbf{Mass}}{\textbf{Density}}$$

$$\text{Volume} = \frac{0.2}{11300}$$

$= 0.000017699 = 1.76699 \times 10^{-5}$

Volume = $1.77 \times 10^{-5} m^3$

(iii) Mass of gold bar = m = 0.2 kg

Density of gold = ρ = 19300 kgm^{-3}

Volume of gold bar = V =?

$$\text{Volume} = \frac{\textbf{Mass}}{\textbf{Density}}$$

$$\text{Volume} = \frac{0.2}{19300}$$

$= 0.000010362 = 1.0362 \times 10^{-5}$

Volume = $1.04 \times 10^{-5} m^3$

7.4 The density of air is 1.3 kgm $^{-3}$. Find the mass of air in a room measuring 8m x 5m x 4m.

(208 kg)

Solution: Density of air = ρ = 1.3 kgm^{-3}

Volume of room = v = 8 m x 5 m x 4 m = 160 m^3

Mass of air = m = ?

Mass of air = Density of air x volume of room

Mass of air = 1.3 x 160

Mass of air = 208 kg

7.5 A student presses her palm by her thumb with a force of 75 N. How much would be the pressure under her thumb having contact area 1.5 cm² ?

($5 x 10^5$ Nm $^{-2}$)

Solution: Force = F = 75 N

Contact Area A = 1.5 cm² = $1.5 \text{ x } \frac{1}{100} \text{ x } \frac{1}{100}$ m² = $1.5 \text{ x } 10^{-4}$ m²

Pressure under the thumb = P = ?

$$P = \frac{F}{A}$$

$$P = \frac{75}{1.5 \times 10^{-4}} = \frac{75}{1.5} \text{ x } 10^4 = 5 \text{ x } 10^5 \text{ Nm}^{-2}$$

7.6 The head of a pin is a square of side 10 mm. Find the pressure on it due to a force of 20 N.

($2 x 10^5$ Nm $^{-2}$)

Solution: Force = F = 20 N

Area of head of a pin A = 10mm x 10mm = $\frac{10}{10}$ cm x $\frac{10}{10}$ cm =

$= \frac{1}{100} \text{ m x } \frac{1}{100} \text{ m}$

$= 10^{-4} \text{ m}^2$

Pressure under the thumb = P = ?

$$P = \frac{F}{A}$$

$$P = \frac{20}{1 \times 10^{-4}} = 2 \text{ x } 10^5 \text{ Nm}^{-2}$$

7.7 A uniform rectangular block of wood 20 cm x 7.5 cm x 7.5 cm and of mass 1000g stands on a horizontal surface with its longest edge vertical. Find

(i) The pressure exerted by the block on the surface

(ii) Density of the wood.

(1778 Nm^{-2} , 889 kgm^{-3})

Solution: Length of the smallest side of the block = 7.5 cm

Mass of the block m = 1000g = 1kg

(i) Pressure exerted by the block P = ?

(ii) Density of wood ρ = ?

Calculations: (i) since the smallest edge of the block is rested on the horizontal surface. Therefore, area of the block will be:

Area = A = 7.5 cm x 7.5 cm = 56.25 cm^2

$$= 56.25 \times \frac{1}{100} \times \frac{1}{100} \; m^2 = 56.25 \times 10^{-4} m^2$$

Pressure under the thumb = P = ?

$$P = \frac{F}{A} = \frac{mg}{A}$$

$$P = \frac{1 \times 10}{56.25 \times 10^{-4}} = 0.1778 \times 10^4 = 1778 \; Nm^{-2}$$

(ii) Volume = V = 20 cm x 7.5 cm x 7.5 cm = 1125 cm^3

$$= 1125 \times \frac{1}{100} m \times \frac{1}{100} m \times \frac{1}{100} m = 1125 \times 10^{-6} m^3$$

Or $V = 1.125 \times 10^{-3} m^3$

Density = $\frac{\textbf{Mass}}{\textbf{Volume}}$

Density = $\frac{1}{1.125 \times 10^{-3}}$ = 0.8888 x 10^3 = 888.8 kgm $^{-3}$

Density = 889 kgm $^{-3}$

7.8 A cube of glass of 5 cm side and mass 306 g, has a cavity inside it. If the density of glass is 2.55 gcm $^{-3}$. Find the volume of the cavity.

(5 cm³)

Solution: Size of the cube = 7.5 cm

Mass of the cube = m = 306 g

Density of glass = ρ = 2.55 kgm $^{-3}$

Volume of the cavity = V =?

Volume of the whole cube = 5 cm x 5 cm x 5 cm = 125 cm^3

Volume of the glass = $\frac{Mass}{Density}$

Volume = $\frac{306}{2.55}$ = 120 cm^3

Volume of the cavity = 125 cm^3 – 120 cm^3 = 5 cm^3

7.9 An object has weight 18 N in air. Its weight is found to be 11.4 N when immersed in water. Calculate its density. Can you guess the material of the object?

(2727 kgm $^{-3}$, Aluminium)

Solution: weight of object in air = w_1 = 18 N

Weight of object immersed in water = w_2 = 11.4 N

Density of glass = ρ = 1000 kgm^{-3}

(i) Density of the object = D = ?

(ii) Nature of the material = ?

(i) $$D = \frac{w_1}{w_1 - w_2} \times \rho$$

$$D = \frac{18}{18 - 11.4} \times 1000$$

$$= \frac{18}{6.6} \times 1000 = 2.727 \times 10^3 = 2727\ kgm^{-3}$$

(ii) The density of aluminum is 2700 kgm^{-3}, the above calculated value of density is 2727 kgm^{-3} nearest to the density of aluminum, so the material of the object is aluminum.

7.10 A solid block of wood of density 0.6 gcm^{-3} weighs 3.06 N in air. Determine (a) volume of the block (b) the volume of the block immersed when placed freely in a liquid of density 0.9 gcm^{-3}?

(510 cm^3 , 340 cm^3)

Solution: Density of wood = D = 0.6 gcm^{-3}

Weight of the wooden block = w= 3.06 N

Since w = mg or $m = \frac{w}{g} = \frac{3.06}{10} = 0.306$ kg = 306 g

Density of liquid D = 0.9 gcm^{-3}

(i) Volume of the block V = ?

(ii) Volume of the block immersed in a liquid V = ?

$$\text{Density} = \frac{Mass}{Volume}$$

$$\text{Volume} = \frac{Mass}{Density}$$

$$V = \frac{306}{0.6} = 510\ cm^3$$

(b) Volume = $\frac{Mass}{Density}$

$$V = \frac{306}{0.9} = 340\ cm^3$$

7.11 The diameter of the piston of a hydraulic press is 30 cm. How much force is required to lift a car weighing 20 000 N on its piston if the diameter of the piston of the pump is 3 cm? (200 N)

Solution: Diameter = D = 30 cm

Radius of the piston = R = $\frac{D}{2} = \frac{30\ cm}{2}$ = 15 cm = $\frac{15}{100}$ m = 0.15 m

Area of the piston = A = $2\pi R^2$ = 2 x 3.14 x $(0.15)^2$

A = 0.1413 m^2

Weight of the car w = F_2 = 20000 N

Diameter of the piston d = 3 cm

Radius of the piston = R = $\frac{D}{2} = \frac{3\ cm}{2}$ = 1.5 cm = $\frac{1.5}{100}$ m = 0.015 m

Area of the piston = A = $2\pi R^2$ = 2 x 3.14 x $(0.015)^2$

A = 1.413 x 10^{-3} m^2

Force = F_1 = ?

$$\frac{F_1}{a} = \frac{F_2}{A}$$

$$F_1 = F_2 \times \frac{a}{A}$$

$$F_1 = 200000\ N \times \frac{1.413 \times 10^{-3}}{0.1413}$$

$$= 200000\ N \times 0.01$$

$$F_1 = 200\ N$$

7.12 A steel wire of cross-sectional area $2x10^{-5}$ m^2 is stretched through 2 mm by a force of 4000 N. Find the Young's modulus of the wire. The length of the wire is 2 m.

($2x10^{11}$ Nm^{-2})

Solution: Cross-sectional area = A = 2×10^{-5} m^2

Extension = ΔL = 2 mm = $2 \times \frac{1}{1000}$ m = 0.002 m

Force = F = 4000 N

Length of the wire = L = 1m

$$Y = \frac{FL}{A\Delta L}$$

$$Y = \frac{4000 \times 2}{2 \times 10^{-3} \times 0.002} = \frac{8000}{0.004 \times 10^{-5}}$$

$$Y = \frac{800}{0.004} \times 10^{-5}$$

Y = 2,000,000 x 10^{-5} = 2 x 10^{11} Nm^{-2}

PROBLEMS

8.1 Temperature of water in a beaker is 50°C, what is its value in Fahrenheit scale?

(122°F)

Solution: Temperature in Celsius scale = C = 50°C

Temperature in Fahrenheit scale = F = ?

$F = 1.8C + 32$

$F = 1.8 \times 50 + 32 = 90 + 32$

$F = 122°F$

8.2 Normal human body temperature is 98.6°F, convert it into Celsius scale and kelvin scale. **(37°C,310K)**

Solution: Temperature in Fahrenheit scale = 98.6°F

(i) Temperature in Celsius scale = ?

(ii) Temperature in Kelvin scale = ?

(i) $F = 1.8C + 32$

$1.8C = F - 32$

$1.8C = 98.6 - 32$

$1.8C = 66.6$

$C = \frac{66.6}{1.8} = 37°C$

(ii) $T(K) = C + 273$

$T(K) = 37 + 273$

$T(K) = 310K$

8.3 Calculate the increase in the length of an aluminum bar 2 m long when heated from 0°C to 20°C, if the thermal coefficient of linear expansion of aluminum is 2.5 × $10^{-5}\,K^{-1}$.
(0.1cm)

Solution: Original length of rod = $L_{0=}$ 2m

Initial temperature = $T_{0=}$ 0°C = 0 + 273 = 273 K

Final temperature = T = 20°C = 20 + 273 = 293 K

Change in temperature = Δ T = T - T_0 = 293 – 273 = 20 K

Coefficient of linear expansion of aluminum = α = 2.5 × $10^{-5}\,K^{-1}$

Increase in volume Δ L = ?

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta L = 2.5 \times 10^{-5} \times 20$$

$$\Delta L = 100 \times 10^{-5}$$

$$\Delta L = 0.001 \text{ m} = 0.001 \times 100 = 0.1\text{cm}$$

8.4 A balloon contains 1.2m^3 air at 15°C. Find its volume at 40°C.Thermal coefficient of volume expansion of air is 3.67× $10^{-3}\,K^{-1}$.

(1.3m^3)

Solution: Original volume = $V_{0=}$ 1.2m^3

Initial temperature = T_0 = 15°C = 15 + 273 = 288 K

Final temperature = T = 40°C = 40 + 273 = 313 K

Change in temperature = Δ T = T - T_0 = 313 – 288 = 25 K

Coefficient of volume expansion of air β = 3.67 × $10^{-3}\,K^{-1}$

Volume = V = ?

$$V = V_0\,(1 + \beta \Delta T)$$

V = 1.2 (1 + 3.67 × 10^{-3} × 25) = 1.2(1+ 91.75 × 10^{-3})

= 1.2(1 + 0.09175) = 1.2× 1.09175

V = 1.3m^3

8.5 How much heat is required to increase the temperature of 0.5 kg of water from 10°C to 65°C?
(115500 J)

Solution: Mass of water = m = 0.5 kg

Initial temperature = T_1 = 10°C = 10 + 273 = 283 K

Final temperature = T_2 = 65°C = 65 + 273 = 338 K

Change in temperature = Δ T = T_2 - T_1 = 338 – 283 = 55 K

Heat = Δ Q = ?

Δ Q = mc Δ T

Δ Q = 0.5× 2400 ×55

Δ Q = 115500J

8.6 An electric heater supplies heat at the rate of 1000J per second. How much time is required to raise the temperature of 200 g of water from 20°C to 90°C?
(58.8 s)

Solution: Power = P = 1000 Js^{-1}

Mass of water = m = 200 g = $\frac{200}{1000}$ = 0.2 kg

Initial temperature = T_2 = 20°C = 20 + 273 = 293 K

Final temperature = T_1 = 90°C = 90 + 273 = 363 K

Change in temperature = Δ T = T_2 - T_1 = 363 – 293 = 70 K

Specific heat of water = c = 4200 $Jkg^{-1}K^{-1}$

Time = t = ?

P = $\frac{w}{t}$

Or P = $\frac{Q}{t}$

Or P × t = Q

Or P × t = mc Δ T

Or t= $\frac{mc\,\Delta\,T}{P}$

t = $\frac{0.2\times 4200\times 70}{1000}$ = 58.8 s

8.7 How much ice will melt by 5000 J of heat? Latent heat of fusion of ice = 336000 J kg^{-1}.
(149 g)

Solution: Amount of heat required to melt ice = 50000J

Latent heat of fusion of ice = = 336000 J kg^{-1}

Amount of ice = m = ?

ΔQ_f = m H_f

Or m = $\frac{\Delta Q_f}{H_f}$

m = $\frac{50000}{336000}$ = 0.1488 kg

= 0.1488 ×1000 = $\frac{1488}{1000}$ ×1000 = 148.8 g ≈ 149 g

8.8 Find the quantity of heat needed to melt 100g of ice at -10°C into water at 10°C.

(39900 J)

(Note: Specific heat of ice is 2100 $Jkg^{-1} K^{-1}$, specific heat of water is 4200 $Jkg^{-1} K^{-1}$, Latent heat of fusion of ice is 336000 Jkg^{-1}).

Solution: Mass of ice = m = 100 g = $\frac{100}{1000}$ = 0.1 kg

Specific heat of ice = c1 = 2100 $Jkg^{-1} K^{-1}$

Latent heat of fusion of ice = L = 336000 Jkg^{-1}

Specific heat of water = c = 4200 $Jkg^{-1} K^{-1}$

Quantity of heat required = Q = ?

Case I:

Heat gained by ice from -10°C to 0°C

$$Q1 = mc\,\Delta T$$

$$Q1 = 0.1\times 2100 \times 10 = 2100\ J$$

Case II:

Heat required for ice to melt = Q2 = mL

$$= 0.1 \times 336000$$

$$Q2 = 33600\ J$$

Case III:

Heat required to raise the temperature of water from 0°C to 10°C

$$Q3 = mc\,\Delta T$$

$$Q3 = 0.1\times 4200 \times 10 = 4200\ J$$

Total heat required = Q = Q1 + Q2 + Q3

$$Q = 2100 + 33600 + 4200$$

$$Q = 39900\ J$$

8.9 How much heat is required to change 100g of water at 100°C into steam? (Latent heat of vaporization of water is $2.26 \times 10^6 Jkg^{-1}$. ($2.26 \times 10^5$J)

Solution: Mass of water = m = 100 g = $\frac{100}{1000}$ = 0.1 kg

Latent heat of vaporization of water = H_v = $2.26 \times 10^6 \ Jkg^{-1}$

Heat required = $\Delta Q_v = ?$

$$\Delta Q_v = mH_v$$

$$\Delta Q_v = 0.1 \times 2.26 \times 10^6 = 0.226 \times 10^6 = \frac{226}{1000} \times 10^6$$

$$= 2.26 \times 10^{-1} \times 10^6 = 2.26 \times 10^5 \text{ J}$$

8.10 Find the temperature of water after passing 5 g of steam at 100°C through 500 g of water at 10°C.
(16.2°C)

(Note: Specific heat of water is 4200 $Jkg^{-1} K^{-1}$, Latent heat of vaporization of water is $2.26 \times 10^6 \ Jkg^{-1}$).

Solution: Mass of stream = m_1 = 5 g = $\frac{5}{1000}$ kg = 0.005 kg

Temperature of stream = T_1 = 100°C

Mass of water = m_2 = 0.5 kg

Temperature of water = T_2 = 10°C

Final temperature = T_3 = ?

Case I:

Latent heat lost by stream = Q1 = mL

Q1 = $0.005 \times 2.26 \times 10^6 = 11.3 \times 10^3$ = 11300 J

Case II:

Heat lost by stream to attain final temperature Q2 = m_1c Δ T

$$Q2 = 0.005\times 4200 \times (100 - T_3)$$

$$Q2 = 21(100 - T_3)$$

Case III:

Heat gained by water Q3 = m_2c Δ T

$$Q3 = 0.5\times 4200 \times (T_3 - 10)$$

$$Q3 = 2100(T_3 - 10)$$

According to the law of heat exchange.

Heat lost by stream = heat gained by water

$$Q1 + Q2 = Q3$$

$$11300 + 21\ (100 - T_3) = 2100\ (T_3 - 10)$$

$$11300 + 2100 - 21T_3 = 2100\ T_3 - 21000$$

$$13400 + 21000 - 21T_3 = 2100T_3 - 21T_3$$

$$34400 = 2121T_3$$

$$T_3 = \frac{34400}{2121}$$

$$T_3 = 16.2\ °C$$

Numerical Problems

9.1. The concrete roof of a house of thickness 20 cm has an area 200 m^2. The temperature inside the house is 15 °C and outside is 35°C. Find the rate at which thermal energy will be conducted through the roof. The value of k for concrete is 0.65 $\mathrm{W\,m^{-1}K^{-1}}$.

(13000 Js^{-1})

Solution: Thickness of the roof = L = 20 cm = $\frac{20}{100}$ = 0.02 m

Area = A = 200 m^2

Temperature outside the house = T_1 = 35°C = 35 + 273 = 308 K

Temperature inside the house = T_2 = 15°C = 15 + 273 = 288 K

Change in temperature = $\Delta T = T_1 - T_2$ = 308 – 288 = 20 K

Value of conductivity for concrete = k = 0.65 W m^{-1} K^{-1}

Rate of conduction of thermal energy = $\frac{Q}{t}$ = ?

$$\frac{Q}{t} = \frac{kA(T_1 \quad T_2)}{L}$$

$$\frac{Q}{t} = \frac{0.65 \times 20 \times 20}{0.02} = \frac{260}{0.02} = 13000 \text{ W}$$

As (1w = 1J s^{-1}) therefore

$$\frac{Q}{t} = 1300 \text{ J s}^{-1}$$

Q9.2 How much heat is lost in an hour through a glass window measuring 2.0 m by 2.5 m when inside temperature is 25 °C and that of outside is 5°C, the thickness of glass is 0.8 cm and the value of k for glass is 0.8 W m – 1 K – 1? (3.6x10 7 J)

Solution: Time = t = 1hour = 3600 s

Thickness of glass = L = 0.8 cm = $\frac{0.8}{100}$ = 0.008 m

Area of a glass window = A = 2.0 m x 2.5 m = 5 m^2

Temperature outside the house = T_1 = 25°C = 25 + 273 = 298 K

Temperature inside the house = T_2 = 5°C = 5 + 273 = 278 K

Change in temperature = $\Delta T = T_1 - T_2$ = 298 – 278 = 20 K

Value of conductivity for concrete = k = 0.8 W m^{-1} K^{-1}

Rate of conduction of thermal energy = $\frac{Q}{t}$ = ?

$$\frac{Q}{t} = \frac{kA(T_1 \quad T_2)}{L}$$

$$Q = \frac{kA(T_1 \quad T_2)}{L} \times t$$

$$\frac{Q}{t} = \frac{0.8 \times 5 \times 20}{0.008} \times 3600 = \frac{80}{0.008} \times 3600 = 36{,}000{,}000 \text{ J} = 3.6 \text{ x } 10^{7} \text{ J}$$

